Fixed and Variable Payout Annuities: How Optimal are “Optimal” Strategies?

Anne MacKay

joint work with Phelim Boyle, Mary Hardy and David Saunders

UNIVERSITY OF WATERLOO

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Outline

1. Variable payout annuities
2. The "classical" optimization problem
3. Exploring new criteria
4. Concluding remarks

"Variable payout" annuities (VPAs): what and why?

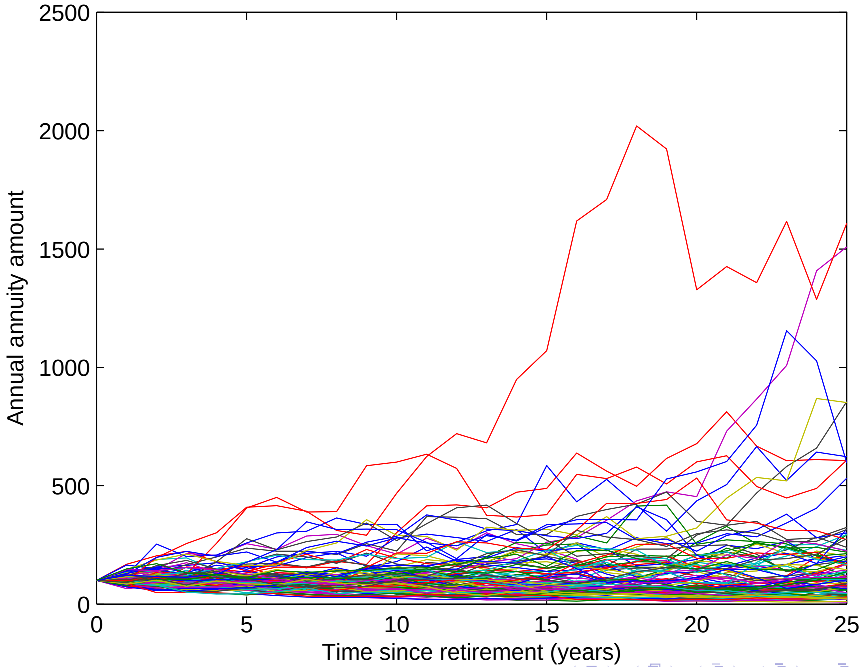
- Group self-annuitization schemes or annuity pools
- Mortality and investment risk retained by the annuitants
- Advantages over traditional fixed annuities:
  - Lower cost (no margins for retained risk)
  - Exposure to equity market (can be a disadvantage)

VPAs in the literature

- Increased interest in the past 10 years (for example, Piggott, Valdez, and Detzel (2005))
- General conclusions:
    - VPAs are preferred when insurers charge for retained risks (Maurer, Mitchell, Rogalla, and Kartashov (2013); Donnelly, Guillén, and Nielsen (2013); Hanewald, Piggott, and Sherris (2013))
    - Small annuitant groups (≈ 100 participants) large enough to provide protection (Donnelly, Guillén, and Nielsen (2013); Stamos (2008))
- Results obtained using CRRA utility maximization

Example of a VPA product

- Initial payment at retirement based on current market and mortality assumptions
- Subsequent payments adjusted periodically
- Adjustment factor $1 + j_t$ reflects investment and mortality experience in past period
- Deficit or surplus in pension fund reflected in annuity payments


2500
2000
1500
1000
500
0
0
5
10
15
20
25
Annual annuity amount
Time since retirement (years)

Setting

- At retirement, the retiree can:
  - Annuitize (VPA or fixed annuity)

$$L_0 = \frac{A_0 \omega_V}{\ddot{a}^V_{0,65}} + \frac{A_0 \omega_F}{\ddot{a}^F_{0,65}} = L_0^V + L^F$$

  - Keep liquid wealth (balanced fund or risk-free asset)
- In subsequent years, the retiree consumes and re-balances her liquid wealth

$$\begin{aligned} L^V_{t+1} &= L^V_t (1 + j_t) \\ L_{t+1} &= L^V_{t+1} + L^F \\ W_{t+1} &= (W_t - C_t)(1 + R(\omega_t)) + L_{t+1} \end{aligned}$$

Function to Maximize

- Retiree seeks to maximize CRRA utility of her consumption

$$\max_{\omega_V, \omega_F, \omega, \mathbf{C}} \quad \sum_{t=0}^{T} \beta^t \; E\left[U(C_t)\right],$$

where $U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$, $\gamma \neq 1$.

- Multi-period optimization can be re-written as a series of one-period optimizations using a Bellman equation

$$H_t(W_t, L_t^V, L_t^F) = \max_{\omega_t, C_t} \left\{ U(C_t) + \beta \; E_t \left[H_{t+1}(W_{t+1}, L_{t+1}^V, L_{t+1}^F) \right] \right\},$$

and

$$H_T(W_T, L_T^V, L_T^F) = U(W_T).$$

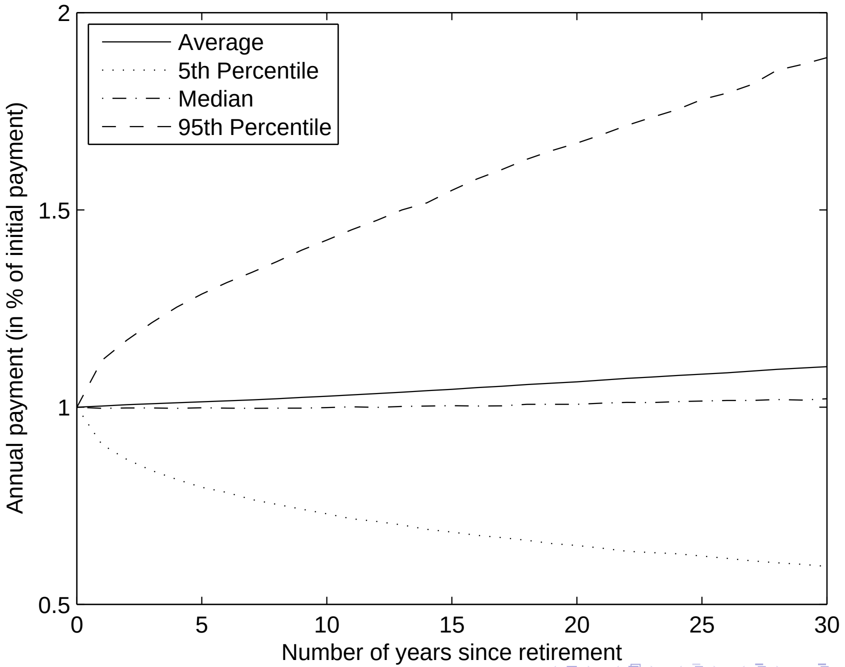
Numerical Results - Key Assumptions

- Retirement age: 65
- Annuitization rate: 0.03
- Risk-free rate: 0.02
- $E[R_t] = 1.06$, $\sqrt{(}Var[R_t]) = 0.2$
- Proportion of VPA fund invested in risky asset: 0.4
- Subjective discount factor β: 0.96
- Risk aversion parameter: 2
- Mortality model: Two-factor model (Cairns, Blake, and Dowd (2006)) with parameters from Maurer, Mitchell, Rogalla, and Kartashov (2013))

Optimal Investment at Retirement

Fixed annuity loading λ	0	0.05	0.07	0.10	0.12
ω_V	0	0.21	0.47	0.82	1
ω_F	1	0.79	0.53	0.18	0
Initial payment (%)	6.95	6.69	6.71	6.83	6.95

Table : Optimal investment strategy at retirement when $\ddot{a}^F_{65,0} = (1+\lambda)\ddot{a}^V_{65,0}$, initial payments as a percentage of wealth at retirement.



Average
5th Percentile
Median
95th Percentile
Annual payment (in % of initial payment)
Number of years since retirement
0.5
1
1.5
2
0
5
10
15
20
25
30

Distribution of Annuity Payments when $\lambda = 0.1$

Age	75	85	95
Mean	1.03	1.06	1.10
Median	1.00	1.01	1.02
5th percentile	0.73	0.65	0.60

Table : Mean, median and 5th percentile of annuity payments as a proportion of the initial payment (6.83% of initial wealth), $\omega_V = 0.82$.

Issues with CRRA utility maximization

- Resulting annuity payments may be too risky for retirees
  - In almost half of the cases, the annuity payment decreases.
  - By age 95, the annuity payment will have decreased by 40% in the worst 5% of cases.

- Consumption is a control variable: does not reflect the reality of a retiree

Two new criteria

- **Poverty threshold:** force annuity payments to remain above a certain level
- **HARA utility function:** reflect habit formation

$$U(C_t) = \frac{(C_t - \overline{C})^{(1-\gamma)}}{1-\gamma},$$

$C_t > \overline{C}$, $\gamma \neq 1$, as used in Kingston and Thorp (2005).

- Assume retiree annuitizes all her wealth and consumes the entire annuity payment, $\lambda = 0.1$ (as in Milevsky (2001)).

Poverty threshold criteria

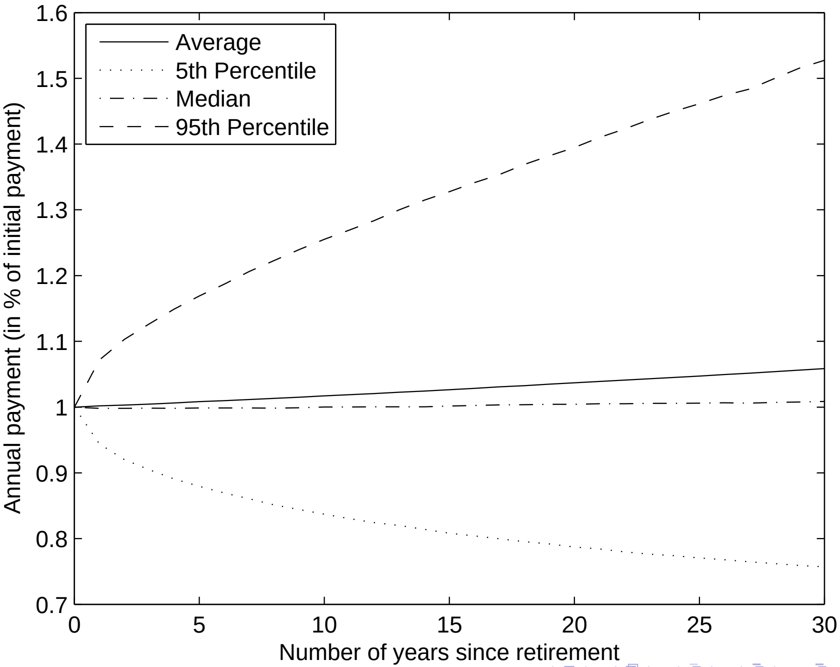
- We want to keep $L_t \geqslant 0.5L_0$ for all L_t, $t = 1, \ldots, T$.

$$\Rightarrow L^F \geqslant 0.5L_0$$

- Set $\omega_F = \frac{\ddot{a}^F_{65}}{\ddot{a}^F_{65} + \ddot{a}^V_{65}}$, $\omega_V = 1 - \omega_F$.

Age	75	85	95
Mean	1.02	1.04	1.06
Median	1.00	1.00	1.01
5th percentile	0.84	0.79	0.76

Table : Mean, median and 5th percentile of annuity payments as a proportion of the initial payment (6.62% of initial wealth), $\omega_V = 0.48$.

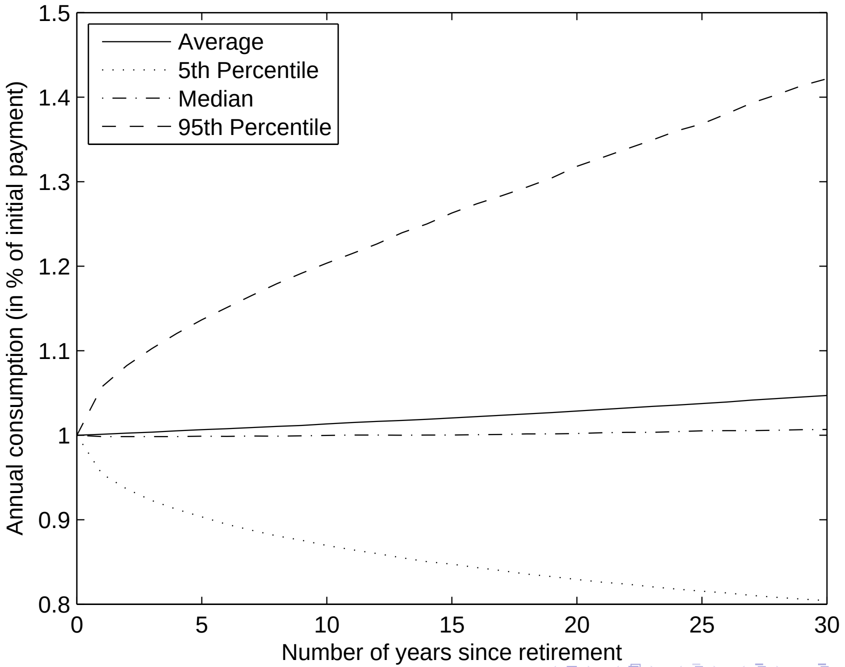

Average
5th Percentile
Median
95th Percentile
Annual payment (in % of initial payment)
Number of years since retirement
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0
5
10
15
20
25
30

Habit formation utility

- Assume $\overline{C} = \frac{0.5A_0}{\ddot{a}^V_{65}} = 0.0347A_0$.
- Using Monte Carlo simulations, obtain ω_V, ω_F that maximize utility.
- Only consider investment strategies that keep $L_t \geqslant \overline{C}$ for all t.

Age	75	85	95
Mean	1.01	1.03	1.05
Median	1.00	1.00	1.01
5th percentile	0.87	0.83	0.80

Table : Mean, median and 5th percentile of annuity payments as a proportion of the initial payment (6.55% of initial wealth), $\omega_V = 0.38$.


Average
5th Percentile
Median
95th Percentile
Annual consumption (in % of initial payment)
Number of years since retirement
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0
5
10
15
20
25
30

Concluding Remarks

- CRRA utility does not necessarily reflect the reality of a retiree
- Variable payout annuities can complement a guaranteed income
- Pooling of mortality risk only (no exposure to equity market) may be interesting for retirees (see Milevsky and Salisbury (2013)).

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